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1987 J. Phys. A: Math. Gen. 20 L113

(http://iopscience.iop.org/0305-4470/20/2/012)

J. Phys. A: Math. Gen. **20** (1987) L113-L116. Printed in the UK

LETTER TO THE EDITOR

Are there long links in percolation backbone in 3D?

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Received 14 July 1986

Abstract. We study the distribution function $f(\mathscr{L})$ for the length of single-connected links for percolation backbone in 3D. We argue that $f(\mathscr{L}) \sim \mathscr{L}^{-\mu} \exp[-(\mathscr{L}/G)^{\gamma}]$, where $\mu = 1.25 \pm 0.05$, $G = 9.5 \pm 1$ and $\gamma = 1.0 \pm 0.1$. We also derive the functional dependence of the distribution of single-connected links in terms of the droplet model of the backbone.

It is well known that the critical effects near the percolation threshold p_c arise from length scales below the correlation length ξ. Bulk properties may be obtained by juxtaposition of pieces of size ξ. So the understanding of the geometrical structure of percolation clusters on scales up to ξ is indispensable in order to investigate physical properties of the percolation system. In this letter we deal with an intrinsic geometrical property of the backbone.

We make use of the following notation: a node is defined as a site of the backbone which has more than two neighbours. The path which connects two nodes and does not pass through another node is defined as a single-connected link (SCL).

In a simple node-link model (Skal and Shklovskii 1974, de Gennes 1976) the backbone at scales less than ξ is a SCL of length $\mathscr{L}$ which scales as $\mathscr{L} \sim \xi^{1/\nu}$. In contrast to this picture in the 'Sierpinski gasket' model of the backbone (Gefen *et al* 1981) there are SCL with microscopic length a_0 only. Coniglio's (1985) modification of the droplet model possesses SCL with lengths a_0 and $8/3a_0$ in 3D. Because the backbone in the droplet model (Sarychev and Vinogradov 1981, 1983a, b) is a random fractal, there exists the distribution of SCL over length. For the random fractal discussed in these papers the SCL distribution function $f(\mathscr{L})$ is proportional to $\mathscr{L}^{-\mu}$.

In this letter the SCL distribution function $f(\mathscr{L})$ is determined numerically for the backbone of a percolation infinite cluster generated on a 3D simple cubic lattice with cyclic boundary conditions. We consider a set of experiments on a $65 \times 65 \times 65$ cubic lattice in which bonds are present at random, with specified density p being uncorrelated. p was chosen to be equal to the percolation threshold $p_c = 0.2492$ (Wilke 1983). We take into account the experiments where the computer program (Hopcroft and Tarjan 1973) has found a biconnected cluster which crosses the sample in any direction. We consider such a biconnected cluster as evidence of the backbone. For each of $N = 960$ realisations we computed the number $M(\mathscr{L})$ of SCL with length $\mathscr{L}$ and finally calculated the distribution function $f(\mathscr{L})$ by averaging $M(\mathscr{L})$ over the total number of results.

The obtained SCL distribution function $f(\mathscr{L})$ indicated in figure 1 is best fitted by the expression

$$f(\mathscr{L}) \sim \mathscr{L}^{-\mu} \exp[-(\mathscr{L}/G)^{\gamma}]. \quad (1)$$

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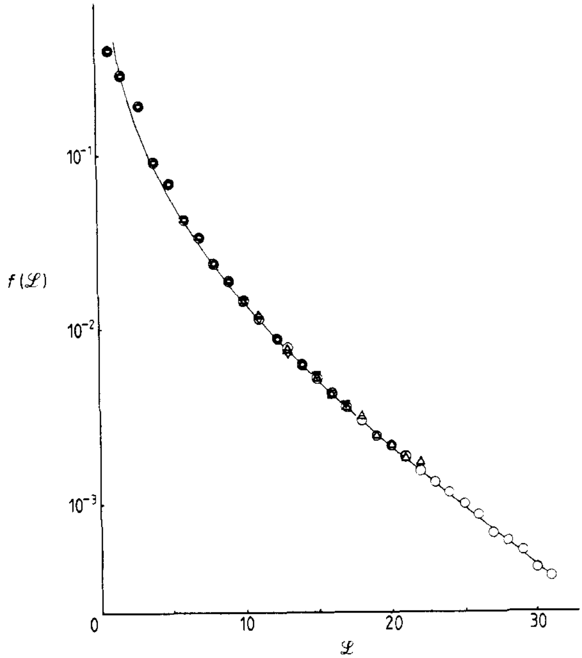

Figure 1. The distribution function $f(\mathscr{L})$ for the length $\mathscr{L}$ of single-connected links for the percolation backbone. ∇, the system size $L=40$; $\triangle$, $L=50$; $\bigcirc$, $L=65$. The size of the symbol is greater than double the standard deviation. The curve represents the best fit (equation (1)) for the system size $L=65$.

By means of a least-square analysis one can find $\mu=1.25\pm0.05$, $G=9.5\pm1.0$ and $\gamma=1.0\pm0.1$.

It is worth emphasising that the characteristic length $G\sim 10a_0$ is of the order of the system size $L=65$. The exponential drop of $f(\mathscr{L})$ may be due to the finite system size so we investigated the values of $\langle\mathscr{L}\rangle$ for systems of size 20, 30, 40 and 50 (see figure 2). We conclude that the mean length of SCL$\langle\mathscr{L}\rangle$ is about $3.5a_0$ and independent

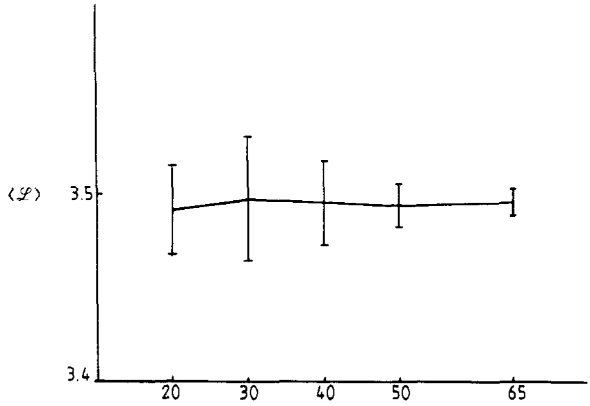

Figure 2. The average length of single-connected links against system size L. The error bars are double the standard deviation.

of the system size L. In this sense there are no long SCL in the backbone. This assumption is confirmed with the relation:

$$\langle \mathscr{L} \rangle = (2N_2+3N_3+4N_4+5N_5+6N_6)/(3N_3+4N_4+5N_5+6N_6) \tag{2}$$

where N_i is the number of backbone sites with i neighbours. Supposing that N_i scales as $L^{d_{f_i}}$ one can obtain

$$\langle \mathscr{L} \rangle \sim L^{d_{\mathscr{L}}} \qquad d_{\mathscr{L}} = \max\{0, d_{f_2} - d_m\} \tag{3}$$

where $d_m = \max\{d_{f_3}, d_{f_4}, d_{f_5}, d_{f_6}\}$. Our calculations result in $d_{f_2} = 1.89 \pm 0.05$, $d_{f_3} = 1.90 \pm 0.1$, $d_{f_4} = 1.85 \pm 0.20$, $d_{f_5} = 1.85 \pm 0.40$, $d_{f_6} = 1.95 \pm 0.80$, $d_f = 1.88 \pm 0.03$ where d_f is a fractal dimension of the backbone. These results support the assumption that $\langle \mathscr{L} \rangle$ is independent of L.

The obtained SCL distribution function $f(\mathscr{L})$ is in disagreement with all the above-mentioned backbone models. The absence of long SCL forces us to modify our droplet model. As a model of an infinite cluster we considered a random self-similar fractal of hierarchical blob-link structure (Sarychev and Vinogradov 1981, 1983a, b). The blob of n hierarchy level consists of $(n-1)$ hierarchy level blobs linked through long SCL. Now we suggest that blobs are linked through SCL of length $\langle \mathscr{L} \rangle$†. There are several ways to form n-level blobs from $(n-1)$-level blobs, but the set of possible configurations is independent of n. Some of these configurations are presented in figure 3.

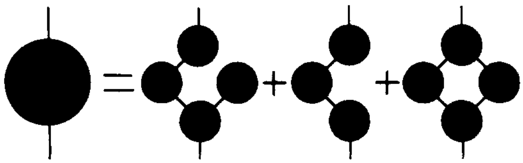

Figure 3. Blob-link structure of a percolation infinite cluster. The blob of 'n' level of hierarchy is presented by three possible configurations of $(n-1)$-level blobs with probabilities p_1, p_2 and p_3, respectively, and so on in a self-similar way.

By using this model one can obtain the qualitative form of the distribution function $f(\mathscr{L})$. For the sake of simplicity we restrict ourselves below only to configurations shown in figure 3.

As is clear from figure 3 (configurations 1, 2) there are single-connected configurations (SCC). For an n-level blob to be a part of a SCL it must consist of SCC at all levels of hierarchy. Hence the probability W_n of an n-level blob to be a part of SCL is governed by the renormalisation group (RG) relation:

$$W_n = (p_1+p_2) W_{n-1}^3 \tag{4}$$

where p_1, p_2 are the probabilities of configurations 1, 2 in figure 3. The solution for equation (4) is

$$W_n = \exp[+0.5 \ln(p_1+p_2) \times 3^n]/(p_1+p_2)^{-1/2}. \tag{5}$$

† It is the blobs that are responsible for the values of critical indices (Sarychev and Vinogradov 1981, 1983a, b). Therefore this assumption cannot change the values. Long SCL were introduced into the model in order to satisfy the Shklovskii theorem (Skal and Shklovskii 1974, Coniglio 1981), but the presence of single-connected configurations of blobs (configurations 1, 2 on figure 3) provides the Shklovskii theorem itself.

The length $\mathscr{L}_n$ of the path through the n-level blob which is a part of SCL obeys the RG relation: $\mathscr{L}_n = 3\mathscr{L}_{n-1}$.

By using this relation repeatedly one can obtain

$$\mathscr{L}_n \sim 3^n. \tag{6}$$

Because of (5) and (6) one can write for the probability W_n

$$W_n \sim \exp[-(\mathscr{L}_n/G)^\gamma] \tag{7}$$

where $\gamma = 1$ (in the more sophisticated droplet models $\gamma \leqslant 1$) and $G \simeq -2/\ln(p_1+p_2)$. Based on the model of the blob-link structure of the backbone it is natural to suggest that the SCL distribution function has an exponential form too. Indeed, the evaluation of $f(\mathscr{L})$ goes as follows. The total length $\mathscr{L}_{\text{tot}}$ of all SCL with lengths greater than $\mathscr{L}_n$ is $\mathscr{L}_n W_n N_n$, where N_n is the number of n-level blobs. It is obvious that $N_{n-1} = N_n(3p_1+3p_2+4p_3)$, where p_3 is a probability of configuration 3 in figure 3. So one can obtain

$$N_n \sim K^{-n} \qquad K = 3p_1+3p_2+4p_3. \tag{8}$$

It follows from (6)-(8) that

$$\mathscr{L}_{\text{tot}} = \mathscr{L}_n W_n N_n \sim \mathscr{L}^{1-\mu} \exp(-\mathscr{L}/G) \tag{9}$$

where $\mu = \ln K/\ln 3$. On the other hand, in terms of the SCL distribution function $f(\mathscr{L})$, the value of $\mathscr{L}_{\text{tot}}$ is proportional to

$$\mathscr{L}_{\text{tot}} \sim \int_{\mathscr{L}}^{\infty} f(x)x\,\mathrm{d}x. \tag{10}$$

Finally one can obtain from (9) and (10)

$$f(\mathscr{L}) \sim \mathscr{L}^{-\mu} \exp(-\mathscr{L}/G). \tag{11}$$

So the droplet model provides a reasonably good fit to numerical data.

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